the reactant gas is 10% butane the H_2 content must be reduced to 63.2%, and the corresponding P_{41} ratio is 37.7.

References

[1] Palmer, H. B. and Knox, B. E., "Contact surface tailoring in a chemical shock tube," ARS J. **31**, 826–828 (1961).

[2] Glass, I. I. and Hall, J. G., "Shock tubes," *Handbook of Supersonic Aerodynamics* (US Government Printing Office, Washington, D. C., 1959), Sec. 18; also NAVORD Rept. 1488, Vol. 6.

Computing Temperature Perturbations on Thin-Skin Panels

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Nomenclature

$\dot{Q}$ = incident heat flux = $\sigma\epsilon_i T_0^4$
T_a = surface temperature = $T_a(r)$
T_0 = radiation equilibrium temperature
T_s = sink temperature
a = skin thickness
k = skin thermal conductivity
k_1 = rivet conductivity
k_2 = insulation conductivity
r_1 = rivet radius
ϵ_i = surface emittance
σ = Stefan-Boltzmann constant

UNDER aerodynamic heating, a time-dependent perturbation in uniform panel-surface temperature appears at the site of a rivet or thermocouple attachment (Fig. 1). Perturbation due to thermocouple attachment is computed by Beck and Hurwicz[1] for the case of an insulated thermocouple sunk in a conduction medium. For a panel constructed of a thin metal skin backed by insulation, or open to radiate to a sink, the perturbation for given incident heat flux and rear-structure temperature is determined at any time. The steady-state heat flow equation is set up in cylindrical coordinates on the skin by setting the incident heat flux less the forward and backward reradiated heat fluxes equal to the net radial heat conduction for an annulus[2]:

$$ak\left(\frac{d^2T_a}{dr^2} + \frac{1}{r}\frac{dT_a}{dr}\right) + \dot{Q} - \sigma\epsilon_i T_a^4 - k_i(T_a - T_s) = 0 \tag{1}$$

where $i = 1$ for $0 \leq r \leq r_1$, and $i = 2$ for $r_1 \leq r \leq r_2$.

The term $k_i(T_a - T_s)$ represents heat conducted to the sink through insulation. If the skin is open to reradiate to a sink, then one approximates this heat by letting $k_i = 4\sigma\epsilon_i T_s^3$ for $T_a \sim T_s$.

The substitutions

$$T_a = T_0 - T_i \qquad T_s = T_0 - T_g \tag{2}$$

will produce a differential equation for the perturbation. T_i is the perturbation and T_g is the temperature difference between radiation equilibrium and the sink. Substituting Eq. (2) in Eq. (1) and dropping terms higher than the first power of T_i in the expansion

$$T_a^4 = T_0^4 - 4T_0^3T_i + 6T_0^2T_i^2 - 4T_0T_i^3 + T_i^4 \tag{3}$$

since $T_i/T_0 \ll 1$,

$$(d^2T_i/dr^2) + (1/r)(dT_i/dr) + \lambda_i^2T_i = B_i\lambda_i^2 \qquad i = 1, 2 \tag{4}$$

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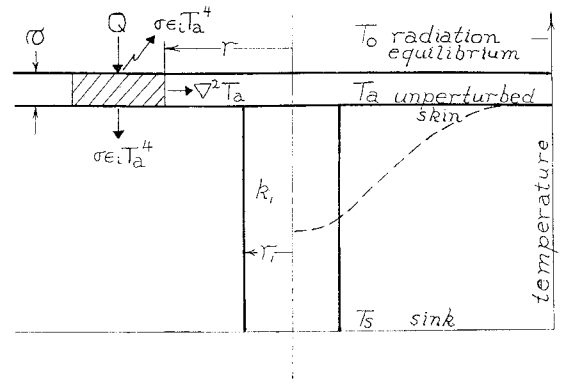

Fig. 1 Thermocouple or rivet attachment to thin-skin panel configuration with a typical temperature perturbation indicated by the dashed line.

where

$$\lambda_i^2 = -(4\sigma\epsilon_i T_0^3 + k_i)/ak \qquad B_i = -k_i T_G/\lambda_i^2 ak \tag{5}$$

The solution to Eq. (4) is of Bessel functions of the first and second kinds with imaginary argument:

$$T_1(r) = AJ_0(j\lambda_1 r) + B_1 \qquad \text{for } 0 \leq r \leq r_1 \tag{6}$$

and

$$T_2(r) = CJ_0(j\lambda_2 r) + DY_0(i\lambda_2 r) + B_2 \qquad \text{for } r_1 \leq r \leq r_2 \tag{7}$$

The boundary conditions are at $r = r_1$, $T_1 = T_2$ and $dT_1/dr = dT_2/dr$, and at $r = r_2$

$$T_2 = -k_2 T_G/(4\sigma_2 T_0^3 + k_2)$$

One assumes $r_2 \geq 10\ r_1$, that is, the perturbation has disappeared at 10 rivet radii. Equations (6) and (7) are solved subject to the boundary conditions simultaneously for the constants. Should $T_1(0)$ be an appreciable fraction of T_0, then an iteration must be performed by including in λ_1^2 the necessary terms of Eq. (3) evaluated for average value of T_1 between $r = 0$ and $r = r_1$ and by iterating Eq. (4) to obtain a new perturbation temperature distribution. The perturbation temperature is large only when $T_s \ll T_0$.

In this case, since the temperature gradient to the sink is assumed linear, one must be sure that the sink temperature is chosen appropriately close to the surface (at the rivet shank for a $\frac{1}{4}$-in.-long rivet) to insure sufficient accuracy.

The perturbation first increases with increasing heat flux, reaches a maximum, and then decreases. This phenomenon occurs whether the sink temperature is constant or follows the surface-temperature increase (Fig. 2).

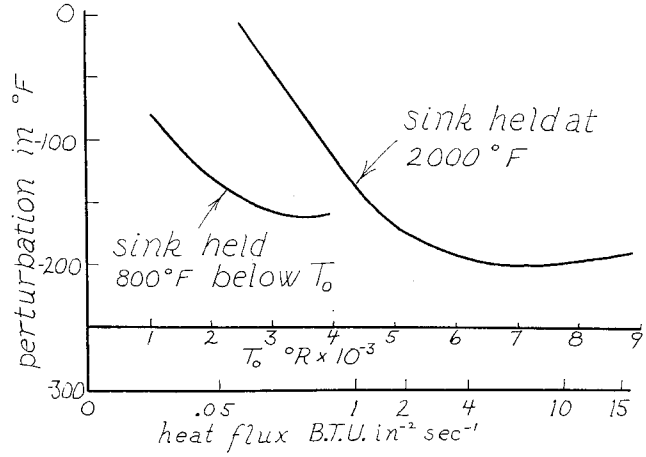

Fig. 2 Temperature perturbation at $r = 0$ as a function of heat flux.

References

[1] Beck, J. F. and Hurwicz, H., J. Heat Transfer **82C**, 27–36 (February 1960).

[2] Schneider, P. J., *Conduction Heat Transfer* (Addison-Wesley Publishing Co., Cambridge, Mass., 1955), 1st ed., pp. 173–175.

Explicit Finite-Difference Method for Calculating Laminar and Turbulent Flows

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THE purpose of this note is to describe an explicit finite-difference program developed at Boeing[1] which has been applied successfully to laminar and turbulent wake and free shear layer problems with finite rate or equilibrium chemistry. Two formulations have been used, one in the physical plane and the other in the von Mises plane. Both are written using forward-difference approximations for the derivatives in x and average-difference approximations for derivatives in r or ψ.

It was found that the physical plane formulation was adequate for the calculation of wakes. However, for shear layer problems, the physical plane formulation had to be abandoned because of stability problems. The von Mises plane formulation proved adequate in this case.

Run times (including print-out) on an IBM 7090 for typical shear layer and wake problems range from 5 to 30 min, depending on the type of chemistry and input flow conditions. It was found that instability usually starts with the energy equation, and the stability parameter is an order of magnitude less than that given by Wu[2] which was derived for the momentum equation. It was found that restrictions placed on step size by convergence and stability considerations were not prohibitive for many flow conditions of interest.

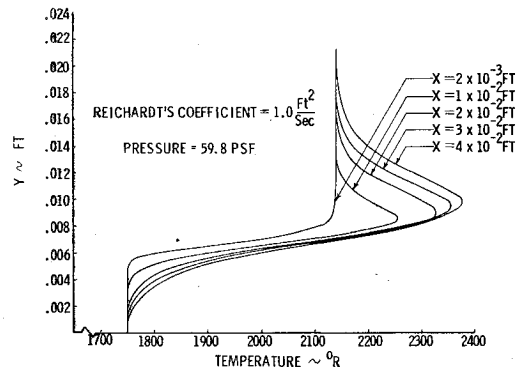

Fig. 1a Thermal profiles for the development of a two-dimensional turbulent mixing zone with combustion.

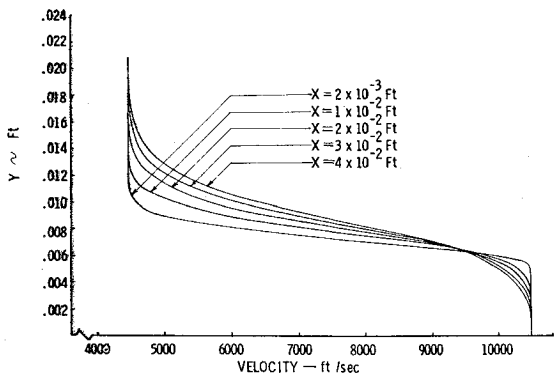

Fig. 1b Velocity profiles for a turbulent, combustible, mixing layer.

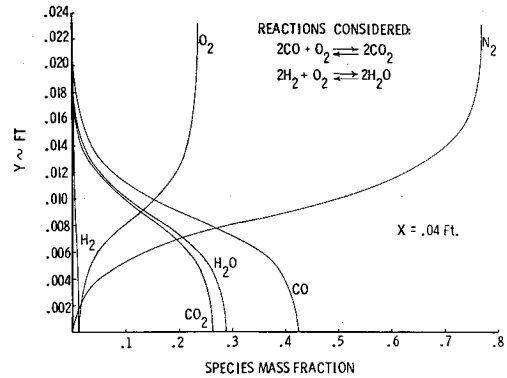

Fig. 1c Species mass fraction profiles for a turbulent mixing layer with combustion.

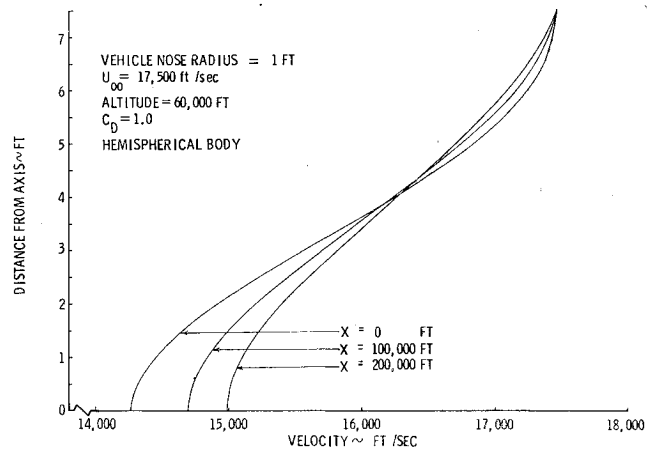

Fig. 2a Velocity profiles for a laminar, equilibrium far wake.

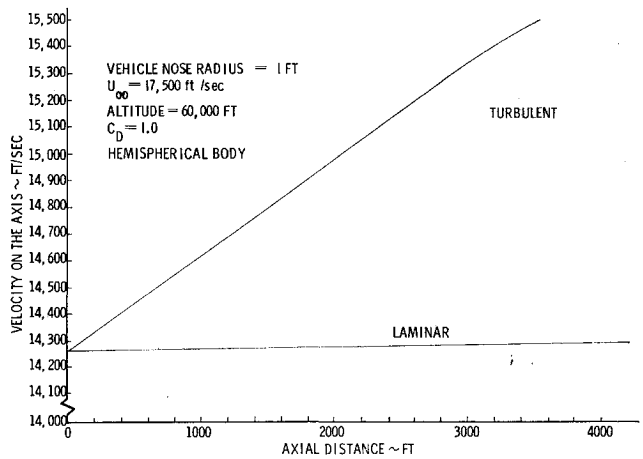

Fig. 2b Comparison of laminar and turbulent axial velocity distributions for an equilibrium wake.

Some typical results are shown in Figs. 1 and 2. Figures 1a–1c present the results of a calculation of the development of a two-dimensional free mixing zone between air and a combustible gas with the following composition (in mass fractions): H_2O, 0.2880; H_2, 0.0124; O_2, 0.0106; CO_2, 0.2630; and CO, 0.4250. Two reactions were considered:

$$2H_2 + O_2 \rightleftarrows 2H_2O$$

$$2CO + O_2 \rightleftarrows 2CO_2$$

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